

AD-A158 112

INSTANTANEOUS ORBIT PARAMETER CHANGES PRODUCED BY  
IMPULSIVE THRUSTING MIT. (U) NAVAL SURFACE WEAPONS  
CENTER DAHLGREN VA A D PARKS FEB 83 NSWC/TR-83-31

1/1

UNCLASSIFIED

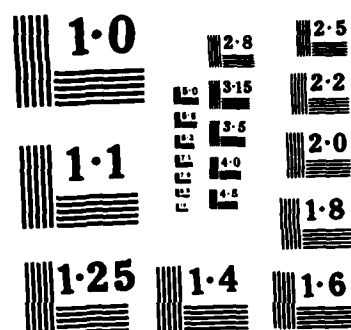
F/G 22/3

NL

END

FILED

END



NATIONAL BUREAU OF STANDARDS  
MICROCOPY RESOLUTION TEST CHART

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 83-31	2. GOVT ACCESSION NO. ADA158112	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INSTANTANEOUS ORBIT PARAMETER CHANGES PRODUCED BY IMPULSIVE THRUSTING WITH APPLICATION TO ORBIT ADJUST DESIGN FOR SATELLITES IN SMALL ECCENTRICITY ORBITS		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. D. Parks		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (K12) Dahlgren, VA 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 35159B; 3K12CS
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Mapping Agency Washington, DC 20305		12. REPORT DATE February 1983
		13. NUMBER OF PAGES 37
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) orbit adjust design small eccentricity orbits Lagrange planetary equations		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An impulsive thrust profile is applied to the Lagrange planetary equations to obtain analytic expressions for thrust-induced changes in orbital parameters. Approximations are introduced for small eccentricity orbits and are used to generate a set of graphical orbit adjust design aids. The orbit adjust design program ORBADJ is discussed, and a program listing and sample output are presented.		

DD FORM 1473  
1 JAN 73EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

## FOREWORD

This document has been prepared to provide a set of graphical tools that may aid the orbit analyst in the selection of thrust parameters to be used during the planning phase of an orbit adjust design for satellites operating in small eccentricity orbits. It is also intended to serve as the formulation document for the NSWC orbit adjust design software program ORBADJ. This is a general software program that may be used to generate thrust parameters for the initial planning phase of an orbit adjust design. This report has been reviewed and approved by Dr. R. J. Anderle.

Released by:

*O. F. Braxton*

O. F. BRAXTON, Head  
Strategic Systems Department



1. TITLE	
2. AUTHOR	
3. PERIODICITY	
4. DATE	
5. PROJECT CODE	
6. PROJECT CODE	
7. PROJECT CODE	
8. PROJECT CODE	
9. PROJECT CODE	
10. PROJECT CODE	
11. PROJECT CODE	
12. PROJECT CODE	
13. PROJECT CODE	
14. PROJECT CODE	
15. PROJECT CODE	
16. PROJECT CODE	
17. PROJECT CODE	
18. PROJECT CODE	
19. PROJECT CODE	
20. PROJECT CODE	
21. PROJECT CODE	
22. PROJECT CODE	
23. PROJECT CODE	
24. PROJECT CODE	
25. PROJECT CODE	
26. PROJECT CODE	
27. PROJECT CODE	
28. PROJECT CODE	
29. PROJECT CODE	
30. PROJECT CODE	
31. PROJECT CODE	
32. PROJECT CODE	
33. PROJECT CODE	
34. PROJECT CODE	
35. PROJECT CODE	
36. PROJECT CODE	
37. PROJECT CODE	
38. PROJECT CODE	
39. PROJECT CODE	
40. PROJECT CODE	
41. PROJECT CODE	
42. PROJECT CODE	
43. PROJECT CODE	
44. PROJECT CODE	
45. PROJECT CODE	
46. PROJECT CODE	
47. PROJECT CODE	
48. PROJECT CODE	
49. PROJECT CODE	
50. PROJECT CODE	
51. PROJECT CODE	
52. PROJECT CODE	
53. PROJECT CODE	
54. PROJECT CODE	
55. PROJECT CODE	
56. PROJECT CODE	
57. PROJECT CODE	
58. PROJECT CODE	
59. PROJECT CODE	
60. PROJECT CODE	
61. PROJECT CODE	
62. PROJECT CODE	
63. PROJECT CODE	
64. PROJECT CODE	
65. PROJECT CODE	
66. PROJECT CODE	
67. PROJECT CODE	
68. PROJECT CODE	
69. PROJECT CODE	
70. PROJECT CODE	
71. PROJECT CODE	
72. PROJECT CODE	
73. PROJECT CODE	
74. PROJECT CODE	
75. PROJECT CODE	
76. PROJECT CODE	
77. PROJECT CODE	
78. PROJECT CODE	
79. PROJECT CODE	
80. PROJECT CODE	
81. PROJECT CODE	
82. PROJECT CODE	
83. PROJECT CODE	
84. PROJECT CODE	
85. PROJECT CODE	
86. PROJECT CODE	
87. PROJECT CODE	
88. PROJECT CODE	
89. PROJECT CODE	
90. PROJECT CODE	
91. PROJECT CODE	
92. PROJECT CODE	
93. PROJECT CODE	
94. PROJECT CODE	
95. PROJECT CODE	
96. PROJECT CODE	
97. PROJECT CODE	
98. PROJECT CODE	
99. PROJECT CODE	
100. PROJECT CODE	

CONTENTS

	Page
INTRODUCTION.....	1
FORMULATIONS .....	1
A SUMMARY OF APPROXIMATIONS VALID FOR SATELLITES IN SMALL ECCENTRICITY ORBITS.....	5
REMARKS CONCERNING PROGRAM ORBADJ.....	11
APPENDIXES:	
A—LISTING OF PROGRAM ORBADJ.....	A-1
B—SAMPLE ORBADJ OUTPUT.....	B-1
DISTRIBUTION.....	(1)

## ILLUSTRATIONS

Figure		Page
1	Change In Semimajor Axis Per Unit In-Track Velocity Impulse .....	7
2	Change In Eccentricity Per Unit Radial Velocity Impulse .....	7
3	Change In Eccentricity Per Unit In-Track Velocity Impulse.....	8
4	Change In Inclination Per Unit Cross-Track Velocity Impulse .....	8
5	Changes In Right Ascension of Ascending Node and True Argument of Latitude Per Unit Cross-Track Velocity Impulse .....	9
6	Changes in Apogee and Perigee Radial Distance Per Unit Radial Velocity Impulse .....	9
7	Change in Apogee Radial Distance Per Unit In-Track Velocity Impulse.....	10
8	Change In Perigee Radial Distance Per Unit In-Track Velocity Impulse .....	10
9	Change In Orbital Period Per Unit In-Track Velocity Impulse.....	11

## INTRODUCTION

The existence of mission-related constraints and flight profiles requires that periodic orbital maintenance thrusting be performed on most artificial earth satellites. This is especially true for low-altitude satellites, since they are continuously perturbed by their interaction with the atmosphere. Operational orbit adjust designs for earth satellites are normally performed in an iterative fashion using coarse satellite ephemerides for early planning and updated precise ephemerides for the final design. Although the desired orbital modifications are known from a comparison of mission requirements with current mission performance, the analyst must still select the most expedient thrust parameters for the design. Initial estimates for these parameters are often made by using the results obtained from orbit perturbation equations and are improved during the iterative design procedure mentioned above.

The following sections of this report are concerned with the development of formulations and data that can be used to provide thrust parameter estimates. Such information can be used as tools for the applied theorist as well as the operational analyst. Considered first are derivations of analytic expressions that relate changes in certain orbital parameters produced by an applied impulsive thrust. These relationships are then used to provide some approximate results that are useful for satellites in small eccentricity orbits. A general discussion of the NSWC orbit adjust design computer program ORBADJ is provided in the final section.

## FORMULATIONS

The changes in orbital parameters caused by instantaneous velocity impulses can be determined from results obtained from the Lagrange planetary equations.<sup>1</sup> These equations, expressed in their Gaussian component form, are given by

$$\dot{a} = 2a^2 \left( \frac{v}{\mu} \right) F_1 \quad (1)$$

---

<sup>1</sup> P. M. Fitzpatrick, *Principles of Celestial Mechanics*, Academic Press, Inc., New York, New York, 1970.

$$\dot{e} = \left( \frac{r \sin \theta}{av} \right) F_R + 2 \left( \frac{\cos \theta + e}{v} \right) F_I \quad (2)$$

$$(i) = \left( \frac{r \cos u}{h} \right) F_C \quad (3)$$

$$\dot{\omega} = - \frac{(1+e^2) \cos \theta + 2e}{e (1 + e \cos \theta)v} F_R + 2 \left( \frac{\sin \theta}{ev} \right) F_I - \frac{r \sin u}{h} \left( \frac{\cos i}{\sin i} \right) F_C \quad (4)$$

$$\dot{\Omega} = \frac{r \sin u}{h} \left( \frac{1}{\sin i} \right) F_C \quad (5)$$

and

$$\dot{u} = - \frac{r \sin u}{h} \left( \frac{\cos i}{\sin i} \right) F_C \quad (6)$$

where  $a$ ,  $e$ ,  $i$ ,  $\omega$ , and  $\Omega$  are the usual Keplerian elements;  $\mu$  is the earth's gravitational constant;  $\theta$  is the true anomaly; and  $u$  is the true argument of latitude defined as

$$u = \theta + \omega \quad (7)$$

The quantities  $r$ ,  $v$ , and  $h$  are the satellite radial distance, speed, and angular momentum, respectively. These are defined as follows:

$$r = \frac{a (1 - e^2)}{1 + e \cos \theta} \quad (8)$$

$$v = \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a} \right)} \quad (9)$$

and

$$h = \sqrt{\mu a (1 - e^2)} \quad (10)$$

The acceleration components  $F_i$  ( $i = R, I, C$ ) appearing in the planetary equations are defined by the vector equation

$$\vec{F} = F_R \hat{R} + F_I \hat{I} + F_C \hat{C} \quad (11)$$

where

$$\hat{C} = \frac{\vec{r} \times \dot{\vec{r}}}{|\vec{r} \times \dot{\vec{r}}|} \quad (12)$$

$$\hat{I} = \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|} \quad (13)$$

and

$$\hat{R} = \hat{I} \times \hat{C} \quad (14)$$

Here  $\vec{r}$  and  $\dot{\vec{r}}$  are the satellite position and velocity vectors.

Verification that the thrust parameters associated with an orbit adjust design produce the desired orbital parameter changes requires that the thrust profile be integrated over the finite thrust interval. A very analytically tractable approach, which yields quite useful estimates, uses an impulsive thrust model in which the Dirac delta function describes the thrust profile. Specifically, if the thrust components defined by

$$F_i = \Delta V_i \delta(t - t_{OA}), (i = R, I, C) \quad (15)$$

where  $\delta$  is the Dirac delta function and  $t_{OA}$  is time at which the thrust impulse is applied, are substituted into Equations 1 through 6 and the equations are integrated over a small time interval centered about  $t_{OA}$ , the following relations are obtained:

$$\Delta a = 2a^2 \left( \frac{v}{\mu} \right) \Delta V_I \quad (16)$$

$$\Delta e = \left( \frac{r \sin \theta}{av} \right) \Delta V_R + 2 \left( \frac{\cos \theta + e}{v} \right) \Delta V_I \quad (17)$$

$$\Delta i = \left( \frac{r \cos u}{h} \right) \Delta V_C \quad (18)$$

$$\Delta \omega = - \frac{(1 + e^2) \cos \theta + 2e}{e(1 + e \cos \theta)v} \Delta V_R + 2 \left( \frac{\sin \theta}{ev} \right) \Delta V_I - \frac{r \sin u}{h} \left( \frac{\cos i}{\sin i} \right) \Delta V_C \quad (19)$$

$$\Delta\Omega = \frac{r \sin u}{h} \left( \frac{1}{\sin i} \right) \Delta V_C \quad (20)$$

$$\Delta u = - \frac{r \sin u}{h} \left( \frac{\cos i}{\sin i} \right) \Delta V_C \quad (21)$$

are all quantities are preadjust values at  $t_{OA}$ .

These results can be applied to those obtained from two-body analytics to obtain results other parameters of interest. The changes in apogee distance, perigee distance, orbital period, and true anomaly are given by

$$\Delta r_A = \left( \frac{r \sin \theta}{v} \right) \Delta V_R + 2 \frac{a}{v} \left( \frac{1+e}{1-e} \right) (1 + \cos \theta) \Delta V_I \quad (22)$$

$$\Delta r_P = - \left( \frac{r \sin \theta}{v} \right) \Delta V_R + 2 \frac{a}{v} \left( \frac{1-e}{1+e} \right) (1 - \cos \theta) \Delta V_I \quad (23)$$

$$\Delta P = 3Pa \left( \frac{v}{\mu} \right) \Delta V_I \quad (24)$$

1

$$\Delta\theta = \frac{(1+e^2) \cos \theta + 2e}{e(1+e \cos \theta)} \Delta V_R - 2 \frac{\sin \theta}{ev} \Delta V_I \quad (25)$$

respectively, where P is the Keplerian period

$$P = 2 \pi \left( \frac{a^3}{\mu} \right)^{1/2} \quad (26)$$

# **A SUMMARY OF APPROXIMATIONS VALID FOR SATELLITES IN SMALL ECCENTRICITY ORBITS**

For sufficiently small eccentricities, the following approximations may be made:

$$r \approx a \quad (27)$$

$$v \approx \sqrt{\frac{\mu}{a}} \quad (28)$$

and

$$h \approx \sqrt{\mu a} \quad (29)$$

By applying these approximations to Equations 16 through 24, one can form the following approximate ratios between thrust components and induced orbital parameter changes:

$$\frac{\Delta a}{\Delta V_I} \approx \frac{P}{\pi} \quad (30)$$

$$\frac{\Delta e}{\Delta V_R} \approx \left(\frac{a}{\mu}\right)^{1/2} \sin \theta \quad (31)$$

$$\frac{\Delta e}{\Delta V_I} \approx 2 \left(\frac{a}{\mu}\right)^{1/2} \cos \theta \quad (32)$$

$$\frac{\Delta i}{\Delta V_C} \approx \left(\frac{a}{\mu}\right)^{1/2} \cos u \quad (33)$$

$$\frac{\Delta \Omega}{\Delta V_C} \sin i \approx \left(\frac{a}{\mu}\right)^{1/2} \sin u \quad (34)$$

$$\frac{\Delta u}{\Delta V_C} \tan i \approx -\left(\frac{a}{\mu}\right)^{1/2} \sin u \quad (35)$$

$$\frac{\Delta r_A}{\Delta V_R} \approx \frac{P}{2\pi} \sin \theta \quad (36)$$

$$\frac{\Delta r_A}{\Delta V_I} \approx \frac{P}{\pi} (1 + \cos \theta) \quad (37)$$

$$\frac{\Delta r_P}{\Delta V_R} \approx -\frac{P}{2\pi} \sin \theta \quad (38)$$

$$\frac{\Delta r_P}{\Delta V_I} \approx \frac{P}{\pi} (1 - \cos \theta) \quad (39)$$

I

$$\frac{\Delta P}{\Delta V_I} \approx 6\pi \left(\frac{a^2}{\mu}\right) \quad (40)$$

ere P is the period given by Equation 26. Ratios for  $\Delta\omega$  and  $\Delta\theta$  are not given because the small divisor problem associated with the small eccentricity assumption. These ratios plotted for the reader's convenience in Figures 1 through 9 for several orbital periods ween 5200 and 7000 sec.

Based on the results given above, the following generalizations may be made:

- Only in-track thrusting can produce changes in a and P.
- Only cross-track thrusting can produce changes in i,  $\Omega$ , and u.
- Radial thrusting produces equal and opposite changes in  $r_A$  and  $r_P$ .
- In-track thrusting at apogee (perigee) produces no change in apogee (perigee) distance, but produces maximum change in perigee (apogee) distance.

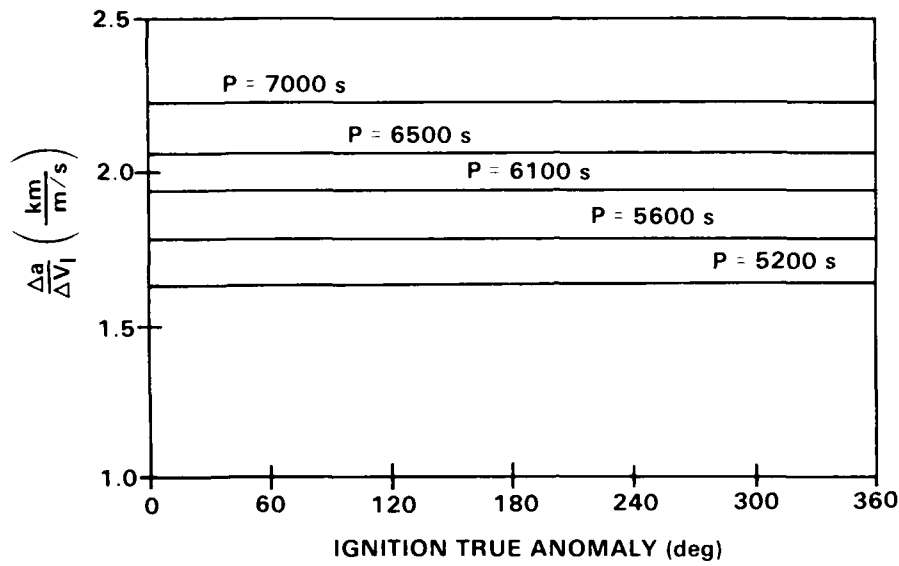


FIGURE 1. CHANGE IN SEMIMAJOR AXIS PER UNIT IN-TRACK VELOCITY IMPULSE

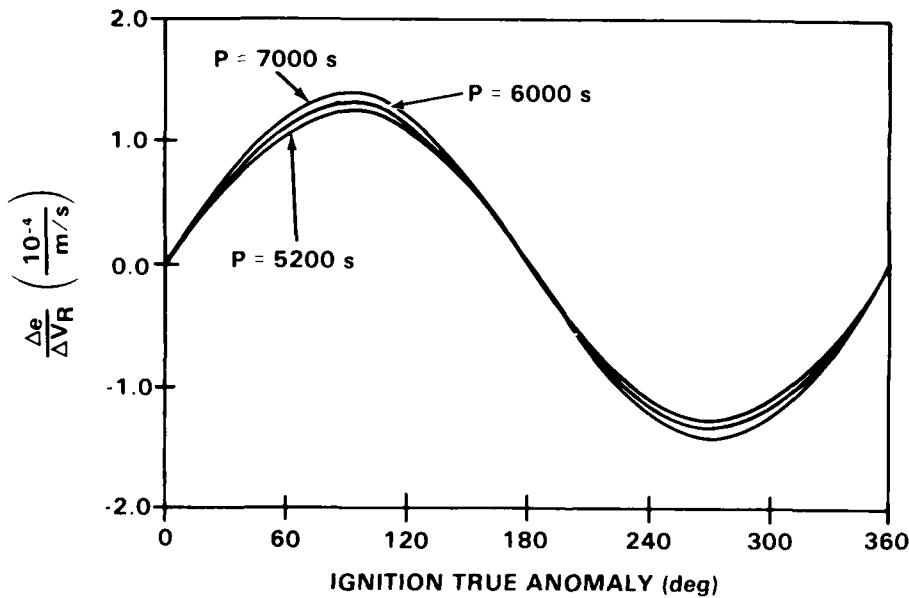


FIGURE 2. CHANGE IN ECCENTRICITY PER UNIT RADIAL VELOCITY IMPULSE

08.39.13 03/11/83

```

PRINT 21,DPC1,CAPC,DVIK,RA
FORMAT(/,5X,*TO PRODUCE A *,F6.2,* SEC PERIOD CHANGE*,/5X,*AND A
1 *,F7.2,* DEG PERIGEE ROTATION REQUIRES AN INTRACK VELOCITY IMPULS
2E OF *,F7.2,* METERS PER SEC*,/5X,* AT A TRUE ANOMALY OF *,F7.2,
3* DEG*)
PRINT 22, J
FORMAT(/,5X,*CONVERGENCE OCCURRED IN *,I5,* ITERATIONS*)
RETURN

PRINT 31, J
FORMAT(/,5X,*CONVERGENCE DID NOT OCCUR IN *,I5,* ITERATIONS*)
STOP
END

SUBROUTINE POSVEL (A,CE,CI,CL,G,H,BO,P1,R2,P3,V1,V2,V3,R,VEL)
CALL NWTRPH (CL,CE,E,SINEP,CCSEP)
HSIN=SIN(H)
HCOS=COS(H)
GSIN=SIN(G)
GCCS=COS(G)
CISIN=SIN(CI)
CICOS=COS(CI)
A11=HCOS*GCCS-HSIN*CICOS*GSIN
A12=-HCOS*GSIN-HSIN*CICOS*GCCS
A21=HSIN*GCCS+HCOS*CICOS*GSIN
A22=HCOS*CICOS*GCCS-HSIN*GSIN
A31=CISIN*GSIN
A32=CISIN*GCCS
FUN=SQRT(1.-CE*CE)
R1=A*(A11*(CCSEP-CE)+A12*(FUN*SINEP))
R2=A*(A21*(CCSEP-CE)+A22*(FUN*SINEP))
R3=A*(A31*(CCSEP-CE)+A32*(FUN*SINEP))
R=A*(1.-CE*CCSEP)
FUN1=(SQRT(BO*A))/R
V1=FUN1*(A11*(-SINEP)+A12*FUN*CCSEP)
V2=FUN1*(A21*(-SINEP)+A22*FUN*CCSEP)
V3=FUN1*(A31*(-SINEP)+A32*FUN*CCSEP)
VEL=FUN1*SQRT(1.-(CE*CCSEP)**2)
RETURN
END

SUBROUTINE NWTRPH (QL,B,E2,SE,CE)
NEWTON RAPHSON SOLUTION TO KEPLERS EQUATION SUBROUTINE
E1=QL+(B*SIN(QL))/(1.-B*COS(QL))
SE=SIN(E1)
CE=COS(E1)
E2=E1+(QL+B*SE-E1)/(1.-B*CE)
IF(ABS(E2-E1)- 1.E-8 )4,4,3
E1=E2
GO TO 2
SE=SIN(E2)
CE=COS(E2)
RETURN
END

```

19

08.39.13 03/11/83

```

PRINT 10
10 FORMAT(///,1X,*CVI(M/S)*,1X,*DVC(M/S)*,1X,*DVR(M/S)*,1X,*DA(KM)*,3
1X,*DE * ,3X,*DI(DEG)*,1X,*DW(DEG)*,1X,*DDM(DEG)*,1X,*DU(DEG)*,1X,
2*DTA(DEG)*,1X,*DRA(KM)*,1X,*DRP(KM )*,1X,*DP(SEC)*,1X,*TAI(DEG)*)

```

```

PRINT 11,DDVI,DCVC,DDVR,DA,DE,DDI,DDW,DDC,DDU,DDTA,DRA,DRP,DP,TTA
11 FORMAT(/,3(1X,F8.3),1X,F6.2,1X,F7.4,1X,F7.4,1X,F7.2,1X,F8.3,1X,
1 F7.2,1X,F8.3,1X,F6.2,1X,F8.2,1X,F7.2,1X,F8.2)

```

```

RETURN

```

```

20 PRINT 21,DA,DE,DDI,DDW,DDC,DDU,DDTA,DRA,DRP,DP,TTA
21 FORMAT(/,28X,F6.2,1X,F7.4,1X,F7.4,1X,F7.2,1X,F8.3,1X,F7.2,1X,F8.3,
1 1X,F6.2,1X,F8.2,1X,F7.2,1X,F8.2)

```

```

RETURN

```

```

END

```

```

SUBROUTINE REQCH(DPCH,DAPC)

```

```

***THIS ROUTINE COMPUTES AN INTRACK VELOCITY IMPULSE REQUIRED TO
***PRODUCE A DESIRED PERIOD CHANGE AND PERIGEE ROTATION.

```

```

COMMON/ELEM/A,E,I,W,O
COMMON/BURN/TAI,DVI,DVC,DVR

```

```

REAL I,MU

```

```

DATA MU/398600.8/,PI/3.14159265359/

```

```

J=1

```

```

DPC=DAPC*PI/180.

```

```

DVC=0.0

```

```

DVR=0.0

```

```

TAI = 0.00

```

```

P = 2.*PI*(((A*A*A)/MU)**0.5)

```

```

10 TAI0 = TAI

```

```

R=(A*(1.-E*E))/(1.+E*COS(TAI))

```

```

V2= MU*((2./R)-(1./A))

```

```

V = SQRT(V2)

```

```

DVI = ( DPC* MU ) / ( 3. * P * A * V )

```

```

SA =0.5*((DPC*E*V)/DVI)

```

```

TAI=ASIN(SA)

```

```

IF(TAI.LT.0.0) TAI=TAI+2.*PI

```

```

TST=ABS(TAI-TAI0)

```

```

IF(TST.LT.0.01) GO TO 20

```

```

J=J+1

```

```

IF(J.GT.500) GO TO 30

```

```

GO TO 10

```

```

20 DVIK=1000.*DVI
BA=TAI*180./PI

```

08.39.13 03/11/83

```

V=SQRT(V2)
U=TAI+H
VP2=(MU/A)*((1.+E)/(1.-E))
VP=SQRT(VP2)
RP=A*(1.-E)
H=VP*RP
IF(U.GT.PI2)U=AMOD(U,PI2)
DRA=((R*SIN(TAI))/V)*DVR+2.*(A/V)*((1.+E)/(1.-E))*(1.+COS(TAI))*
1   DVI

DRP=-((R*SIN(TAI))/V)*DVR+2.*(A/V)*((1.-E)/(1.+E))*(1.-COS(TAI))*
1   DVI

DP=3.*P*A*(V/MU)*DVI

DA=2.*A*A*(V/MU)*DVI

DE=((R*SIN(TAI))/(A*V))*DVR+2.*((COS(TAI)+E)/V)*DVI

DI=((R*COS(L))/H)*DVC

DW=-(((1.+E)*COS(TAI)+2.*E)/(E*(1.+E*COS(TAI))*V))*DVR+2.*(SIN(T
1   AI)/(E*V))*DVI-((R*SIN(U)*COS(I))/(H*SIN(I)))*DVC

DOM=((R*SIN(U))/(H*SIN(I)))*DVC

DU=-((R*SIN(L)*COS(I))/(H*SIN(I)))*DVC

DTA=DU-DW

RETURN
END
SUBROUTINE CLTD(J)

```

\*\*\*\*THIS PROGRAM OUTPUTS THE THRUST INDUCED CHANGES TO THE ORBITAL  
 \*\*\*\*PARAMETERS.

```

COMMON/BURN/TAI,DVI,DVC,DVR
COMMON/DELTA/DRA,DRP,DP,DA,DE,DI,DW,DOM,DU,DTA
DATA PI/3.14159265359/

```

```

DDVI=1000.*DVI
DDVC=1000.*DVC
DDVR=1000.*DVR
DDI =DI*180./PI
DDW=DW*180./PI
DDO=DOM*180./PI
DDL=DU*180./PI
DDTA=DTA*180./PI
TTA=TAI*180./PI

```

```

IF(J.NE.1) GO TO 20
PRINT 9
9 FORMAT(////,5X,*THRUST PARAMETERS AND INDUCED CHANGES IN ORBITAL P
  1PARAMETERS*)

```

08.39.13 03/11/83

```

3      *ARGUMENT OF PERIGEE      **G16.10,*DEG*/5X,
4      *RIGHT ASCENSION OF ASCENDING NODE**G16.10,*DEG*/5X,
5      *TRUE ANOMALY             **G16.10,*DEG*)

```

```

RETURN
END
SUBROUTINE PNAH(TAI,E,AM)

```

```

*****THIS ROUTINE COMPUTES AN APPROXIMATE VALUE FOR THE MEAN
*****ANOMALY ASSOCIATED WITH A GIVEN TRUE ANOMALY TO THIRD
*****ORDER IN ECCENTRICITY.

```

```

      AM=TAI-2.*E*SIN(TAI)+0.75*E*E*SIN(2.*TAI)-0.33333*E*E*E*SIN(3.*
1      TAI)

```

```

RETURN
END
SUBROUTINE CUTC(X,Y,Z,XD,YD,ZD,R,V)

```

```

*****THIS ROUTINE OUTPUTS CARTESIAN POSITION AND VELOCITY COMPONENTS,
*****AS WELL AS THE ASSOCIATED RADIUS AND SPEED.

```

```

      PRINT 10,X,Y,Z,XD,YD,ZD,R,V
10 FORMAT(/,10X,*X      **G16.10,*KM*/10X,
1      *Y      **G16.10,*KM*/10X,
1      *Z      **G16.10,*KM*/10X,
1      *XDUT   **G16.10,*KM PER SEC*/10X,
1      *YDCT   **G16.10,*KM PER SEC*/10X,
1      *ZDCT   **G16.10,*KM PER SEC*/10X,
1      *RADIUS **G16.10,*KM*/10X,
1      *SPFED  **G16.10,*KM PER SEC*)

```

```

RETURN
END
SUBROUTINE ADJUST

```

```

*****THIS COMPUTES CHANGES TO ORBITAL PARAMETERS DUE TO IMPULSIVE
*****VELOCITY CHANGES.

```

```

COMMON/ELEM/A,E,I,W,Q
COMMON/BURN/TAI,CVI,CVC,DVR
COMMON/DELTA/DPA,DPP,DP,DA,DE,DI,DW,DCM,DU,DTA

```

```

REAL I,MU

```

```

DATA MU/398600.6/,PI/3.14159265359/

```

```

      R=A*(1.-F*E)/(1.+F*COS(TAI))

```

```

      P=2.*PI*(((A*A*A)/MU)**.5)
      PI2=2.*PI
      V2=ML*((2./R)-(1./A))

```

08.39.13 03/11/83

```

      V=VBR+(J-1)*VGR
      IF(IDC.EQ.0)GO TO 121
      IF(IDC.EQ.1)GO TO 122
      DVR=V
      DVI=0.0
      DVC=0.0
      GO TO 123
121   DVR=V
      DVI=0.0
      DVC=0.0
      GO TO 123
122   DVC=V
      DVR=0.0
      DVI=0.0
123   CONTINUE
C
C
      DO 130 K=1,IJA
      TAI=AB+(K-1)*AG
C
      CALL ADJUST
C
      CALL OUTD(K)
C
130   CONTINUE
120   CONTINUE
C
C
      STOP
C
C
C*****ENTER MODE=2 PROCESSING
C
C
200   PRINT 10
      PRINT 210
210   FORMAT(/ ,1X,*MODE=2 ORBIT ADJUST DESIGN*)
C
C
      READ *,DPCH,DAPC
C
      CALL REQCH(DPCH,DAPC)
C
      GO TO 220
C
      END
      SUBROUTINE OUTK(A,E,INC,PER,RA,ANOM)
      REAL INC
C
C*****THIS ROUTINE OUTPUTS KEPLERIAN ELEMENTS
C
      PRINT 10,A,E,INC,PER,RA,ANOM
10   FORMAT (/ ,5X,*SEMI MAJOR AXIS
1     *ECCENTRICITY
2     *INCLINATION
      * ,G16.10,*KM*/5X,
      * ,G16.10/5X,
      * ,G16.10,*DEG*/5X,

```

08.39.13 03/11/83

```

      CALL ADJUST
C
      CALL OUTD(1)
C
      PRINT 14
14  FORMAT(///,5X,*PCST-ADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTOR
      1S*)
C
      A=A+DA
      E=E+DE
      INC=INC+DI*180./PI
      I=I+DI
      PER=PER+Dh*180./PI
      W=W+Dh
      RA=RA+DOM*180./PI
      O=O+DOM
      ANOM=ANOM+DTA*180./PI
      TAI=TAI+DTA
C
      CALL OUTK(A,E,INC,PER,RA,ANOM)
C
      CALL MNAN(TAI,E,AM)
C
      CALL POSVEL(A,E,I,AM,W,O,MU,X,Y,Z,XD,YD,ZD,R,V)
C
      CALL OUTC(X,Y,Z,XD,YD,ZD,R,V)
C
      STOP
C
C
C****BEGIN MODE=1 PROCESSING
C
      100 READ *,ANB,ANE,ANG,IDC,VB,VE,VG
C
C****CONVERT TO RADIAN MEASURE AND KM. PER SEC.
C
      AB=ANB*PI/180.
      AE=ANE*PI/180.
      AG=ANG*PI/180.
C
      VBR=VB/1000.
      VER=VE/1000.
      VGR=VG/1000.
C
C
      PRINT 10
      PRINT 110
110  FORMAT(///,1X,*MODE=1 ORBIT PARAMETER CHANGE ENVELOPES*)
      PRINT 10
C
      IJA=((AE-AB)/AG)+2
      IJV=((VER-VBR)/VGR)+2
C
      DO 120 J=1,IJV

```

08.39.13 03/11/83

```

C *****
C
C
COMMON/ELEM/A,E,I,W,O
COMMON/BURN/TAI,DVI,DVC,DVR
COMMON/DELTA/DRA,DRP,DP,DA,DE,DI,DW,DOM,DU,DTA
C
REAL INC,I,ML
C
DATA PI/3.14159265359/,MU/398600.8/
C
C
C
READ *, MODE
READ *, A,E,INC
READ *, PER,RA
C
C*****CONVERT DEGREES TO RADIANS
C
I=INC*PI/180.
W=PER*PI/180.
O=RA*PI/180.
C
C*****SELECT PROPER MODE LOGIC
C
IF(MODE.EQ.1) GO TO 100
IF(MODE.EQ.2) GO TO 200
C
C*****ENTER THE MODE=0 PROCESSING
C
READ *,ANOM,CELI,DELC,DELR
C
TAI=ANOM*PI/180.
DVI=CELI/1000.
DVC=DELC/1000.
DVR=DELR/1000.
C
PRINT 10
10 FORMAT(1H1)
PRINT 11
11 FORMAT(///,1X,*MODE=0 ORBIT ADJUST DESIGN*)
C
220 PRINT 12
12 FORMAT(///,5X,*PREADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTORS*
1)
CALL OUTK(A,E,INC,PER,RA,ANOM)
C
CALL MNAN(TAI,E,AM)
C
CALL POSVEL(A,E,I,AM,W,C,MU,X,Y,Z,XD,YD,ZD,R,V)
C
CALL OUTC(X,Y,Z,XD,YD,ZD,R,V)
C
C*****COMPUTE THRUST INDUCED CHANGES TO ORBITAL PARAMETERS
C

```

08.39.13 03/11/83

PROGRAM ORBADJ (INPUT,CUTPUT,TAPE6=OUTPUT)

```

*****
*
* THIS PROGRAM COMPUTES THE CHANGES PRODUCED IN KEPLERIAN
* ELEMENTS AND OTHER RELATED ORBITAL PARAMETERS DUE TO IMPULSIVE
* THRUSTING. THREE COMPUTATIONAL MODES ARE AVAILABLE AND ARE
* DELINEATED IN THE INPUT CARD STRUCTURE BELOW:
*
*
* CARD 1: COMPUTATIONAL MODE FLAG (MODE). FOR
*   MODE=0   ORBITAL PARAMETER CHANGES AND PRE- AND POST-
*             ORBIT ADJUST KEPLERIAN ELEMENTS AND THE
*             ASSOCIATED CARTESIAN POSITION AND VELOCITY
*             COMPONENTS ARE COMPUTED BASED UPON A USERS
*             DESIGN.
*   MODE=1   ORBITAL PARAMETER CHANGES ARE COMPUTED FOR A
*             USER SPECIFIED DELTA-VELOCITY RANGE AND
*             GRANULARITY AND IGNITION TRUE ANOMALY RANGE
*             AND GRANULARITY.
*   MODE=2   AN INTRACK DELTA-VELOCITY AND IGNITION TRUE
*             ANOMALY ARE COMPUTED FOR A USER SPECIFIED
*             PERIOD AND ARGUMENT OF PERIGEE CHANGE. ALSO
*             COMPUTED ARE THE ASSOCIATED CHANGES IN THE
*             ORBITAL PARAMETERS.
*
* CARD 2: FIRST KEPLERIAN ELEMENT CARD CONTAINING IN FREE FORMAT
*         THE SEMI-MAJOR AXIS IN KILOMETERS, THE ECCENTRICITY,
*         AND THE INCLINATION IN DEGREES.
*
* CARD 3: SECOND KEPLERIAN ELEMENT CARD CONTAINING IN FREE FORMAT
*         THE ARGUMENT OF PERIGEE AND RIGHT ASCENSION OF THE
*         ASCENDING NODE BOTH EXPRESSED IN DEGREES.
*
* FOR MODE=0 ONLY :
*
* CARD 4: IGNITION TRUE ANOMALY IN DEGREES, IN-TRACK VELOCITY
*         IMPULSE IN METERS PER SECOND, CROSS-TRACK VELOCITY
*         IMPULSE IN METERS PER SECOND, AND RADIAL VELOCITY
*         IMPULSE IN METERS PER SECOND .
*
* FOR MODE=1 ONLY :
*
* CARD 4: STARTING IGNITION TRUE ANOMALY, STOPPING IGNITION TRUE
*         ANOMALY, IGNITION TRUE ANOMALY GRANULARITY ALL IN
*         DEGREES; IMPULSE DIRECTION CODE (=0 FOR IN-TRACK,=1
*         FOR CROSS-TRACK,=2 FOR RADIAL); ASSOCIATED STARTING
*         IMPULSE, STOPPING IMPULSE, AND IMPULSE GRANULARITY
*         (ALL IN METERS PER SECOND).
*
* FOR MODE=2 ONLY :
*
* CARD 4: THE DESIRED PERIOD CHANGE IN SECONDS; THE DESIRED
*         CHANGE IN ARGUMENT OF PERIGEE IN DEGREES.

```

NSWC TR 83-31

**APPENDIX A**  
**LISTING OF PROGRAM ORBADJ**

impulse range and granularity and ignition true anomaly range and granularity. The last mode computes the in-track velocity impulse and ignition true anomaly required to produce a user-supplied period change and argument of perigee rotation. Changes in the orbital parameters are also computed, as well as the associated preadjust and postadjust Keplerian elements and Cartesian position and velocity components. A listing of this program is provided in Appendix A. Samples of ORBADJ output are presented in Appendix B.

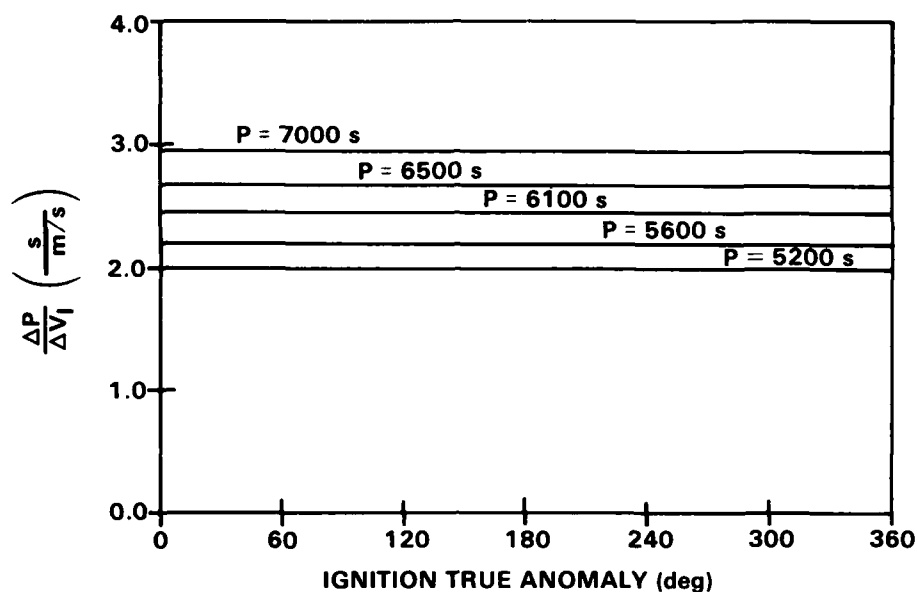


FIGURE 9. CHANGE IN ORBITAL PERIOD PER UNIT IN-TRACK VELOCITY IMPULSE

To illustrate the utility of Figures 1 through 9, consider the simple example where an orbit adjust must be designed for a low-altitude satellite operating in a 5200-s orbit. It is required that this adjust increase the period by 40 s with no corresponding change in perigee distance. From Figure 9 it is seen that an in-track velocity impulse of approximately 20 m/s applied at any true anomaly will increase the period by 40 s. Since no change in perigee distance is wanted, the thrust must be applied at perigee (i.e., a true anomaly of  $0^\circ$ ). However, as can be seen from Figures 3 and 7, the eccentricity and apogee distance will be increased by about  $5.1 \times 10^{-3}$  and 66 km, respectively. Since cross-track thrusting is not applied, the inclination, right ascension of the ascending node, and true argument of latitude will not be changed.

#### REMARKS CONCERNING PROGRAM ORBADJ

The exact impulsive thrust equations of the Formulations section have been implemented into an NSWC computer program called ORBADJ. This program can be used to provide initial orbit adjust designs and can operate in one of three different user selectable modes. The first mode computes orbital parameter changes, pre- and post-orbit adjust Keplerian elements, and the associated Cartesian position and velocity components based on a user-supplied design. The second mode computes orbital parameter changes based on a user-supplied velocity

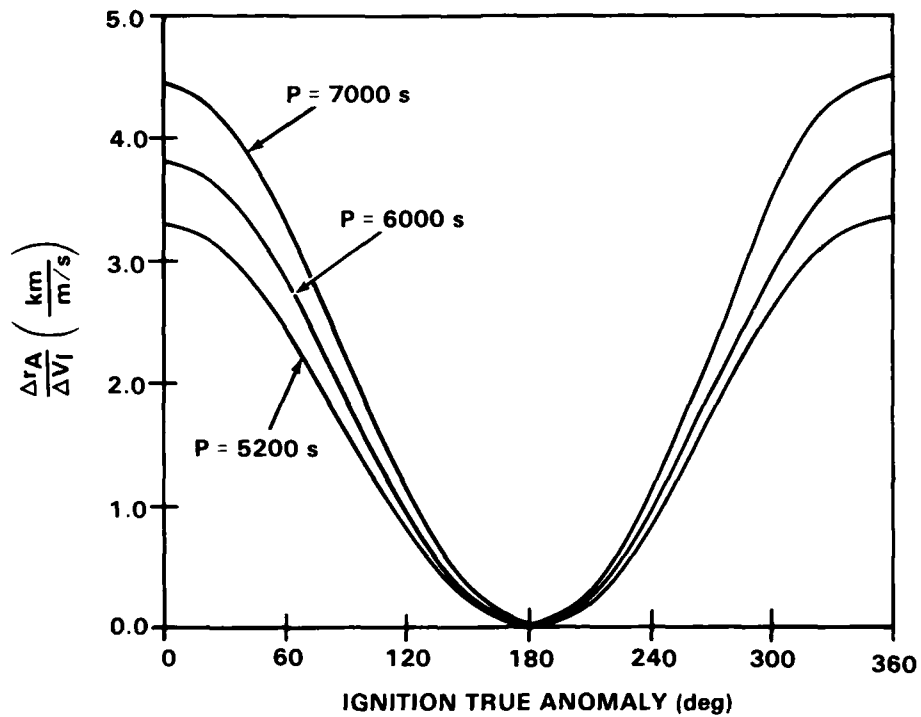


FIGURE 7. CHANGE IN APOGEE RADIAL DISTANCE PER UNIT IN-TRACK VELOCITY IMPULSE

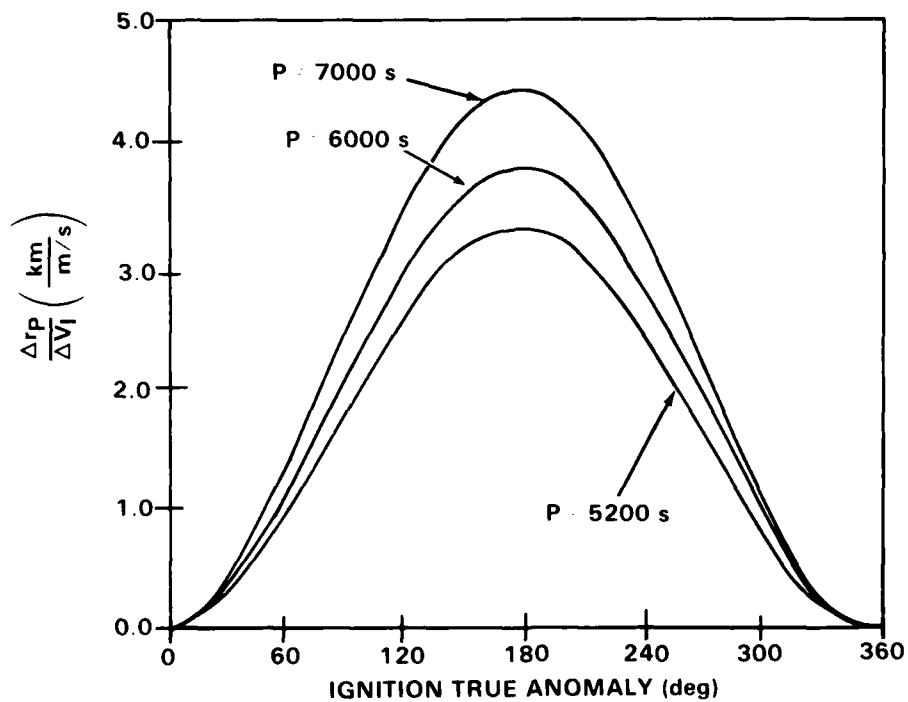


FIGURE 8. CHANGE IN PERIGEE RADIAL DISTANCE PER UNIT IN-TRACK VELOCITY IMPULSE

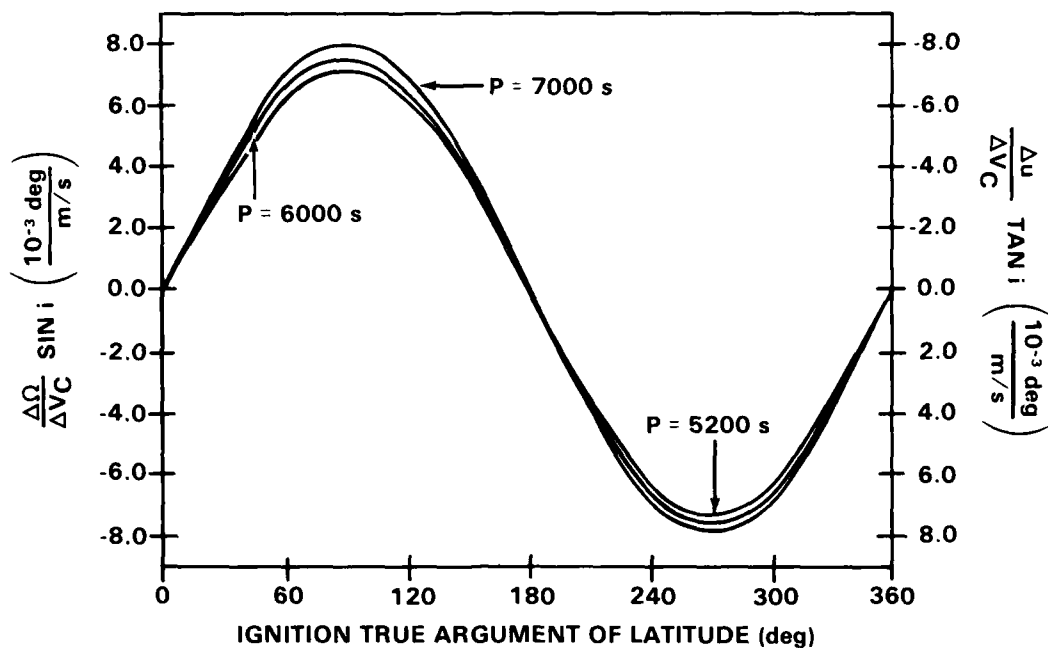


FIGURE 5. CHANGES IN RIGHT ASCENSION OF ASCENDING NODE AND TRUE ARGUMENT OF LATITUDE PER UNIT CROSS-TRACK VELOCITY IMPULSE

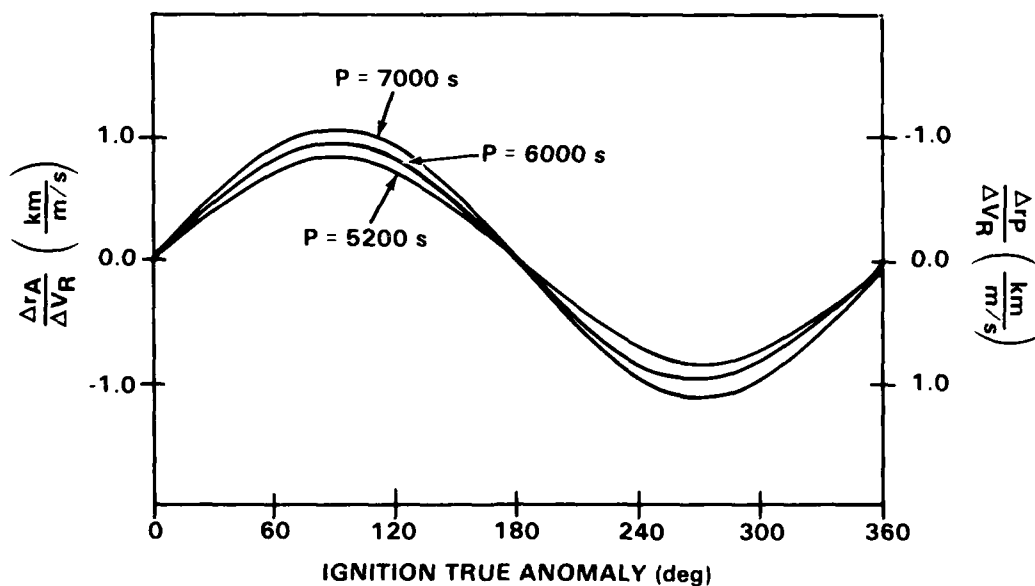


FIGURE 6. CHANGES IN APOGEE AND PERIGEE RADIAL DISTANCE PER UNIT RADIAL VELOCITY IMPULSE

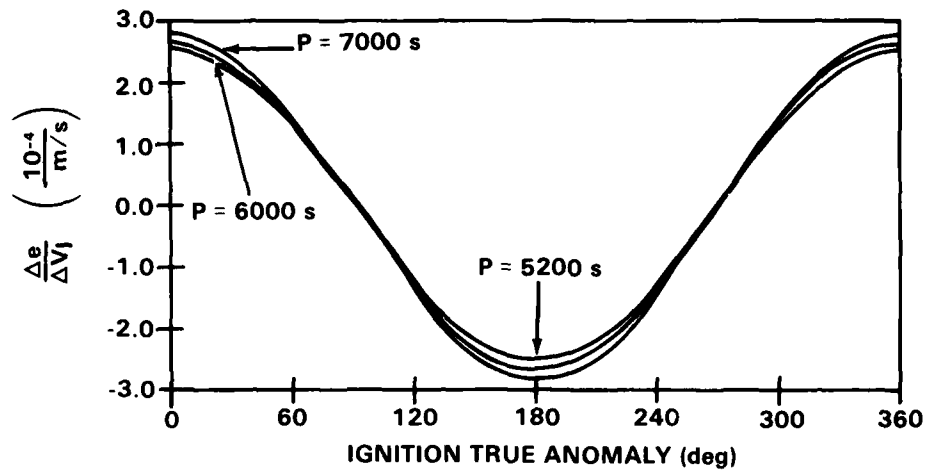


FIGURE 3. CHANGE IN ECCENTRICITY PER UNIT IN-TRACK VELOCITY IMPULSE

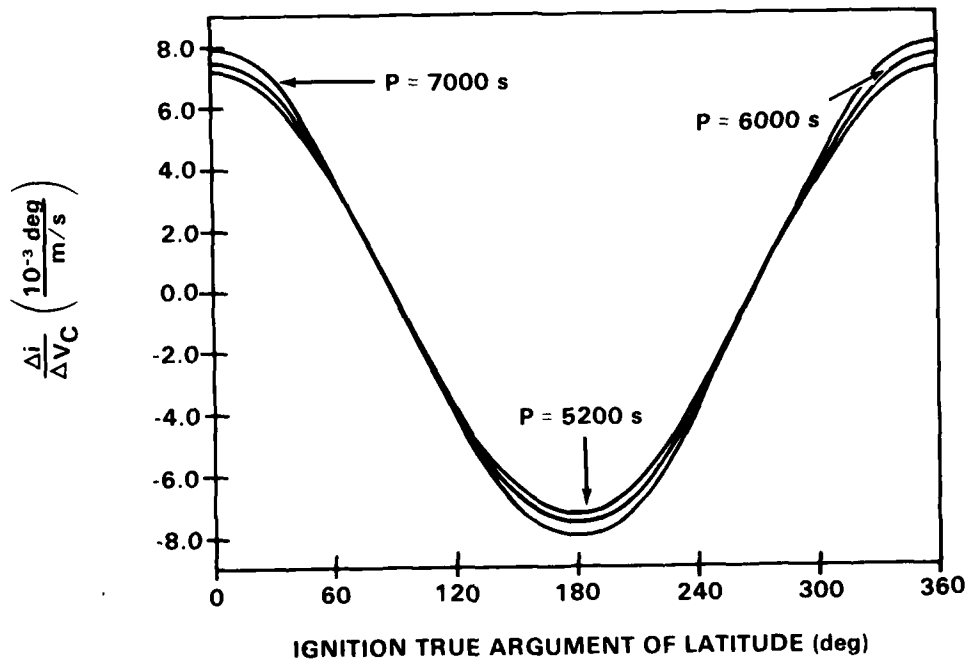


FIGURE 4. CHANGE IN INCLINATION PER UNIT CROSS-TRACK VELOCITY IMPULSE

NSWC TR 83-31

APPENDIX B

SAMPLE ORBADJ OUTPUT

MODE=0 ORBIT ADJUST DESIGN

PREADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTORS

SEMAJOR AXIS = 6756.776000 KM  
 ECCENTRICITY = .1810700000E-01  
 INCLINATION = 96.9740000 DEG  
 ARGUMENT OF PERIGEE = 178.3081000 DEG  
 RIGHT ASCENSION OF ASCENDING NODE = 336.0182000 DEG  
 TRUE ANOMALY = 0.

X = -6068.735922 KM  
 Y = 2673.633495 KM  
 Z = 194.4317197 KM  
 XDOT = .1748423179 KM PER SEC  
 YDOT = .9011706505 KM PER SEC  
 ZDOT = -7.759773932 KM PER SEC  
 RADIUS = 6634.431057 KM  
 SPEED = 7.821030005 KM PER SEC

B-3

THRUST PARAMETERS AND INDUCED CHANGES IN ORBITAL PARAMETERS

DVL(M/S)	DVC(M/S)	DVR(M/S)	DA(KM)	DE	DI(DEC)	DN(DEC)	DOM(DEC)	DUI(DEC)	DTA(DEC)	DRA(KM)	DRP(KM)	DP(SEC)	TAI(DEC)
5.000	0.000	0.000	0.96	.0013	0.0000	0.00	0.000	0.00	0.000	17.92	0.00	10.99	0.00

POST-ADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTORS

SEMAJOR AXIS = 6765.733872 KM  
 ECCENTRICITY = .1940875565E-01  
 INCLINATION = 96.9740000 DEG  
 ARGUMENT OF PERIGEE = 178.3081000 DEG  
 RIGHT ASCENSION OF ASCENDING NODE = 336.0182000 DEG  
 TRUE ANOMALY = 0.

X = -6068.725255 KM  
 Y = 2673.620756 KM  
 Z = 194.4313780 KM  
 XDOT = .1749542130 KM PER SEC  
 YDOT = .9017857777 KM PER SEC  
 ZDOT = -7.764740013 KM PER SEC  
 RADIUS = 6634.419396 KM  
 SPEED = 7.826035285 KM PER SEC

## THRUST PARAMETERS AND INDUCED CHANGES IN ORBITAL PARAMETERS

DVI(M/S)	DVC(M/S)	DVR(M/S)	DA(KM)	DE	DI(DEG)	DM(DEG)	DNM(DEG)	DU(DEG)	DTA(DEG)	DRA(KM)	DRP(KM)	DP(SEC)	VAI(DEG)
2.000	0.000	0.000	3.58	.0005	0.0000	0.00	0.000	0.00	0.000	7.17	0.00	4.40	0.00
		0.000	3.58	.0005	0.0000	.42	0.000	0.00	-.419	7.05	.11	4.39	15.00
		0.000	3.57	.0005	0.0000	.81	0.000	0.00	-.811	6.70	.45	4.39	30.00
		0.000	3.56	.0004	0.0000	1.15	0.000	0.00	-1.150	6.15	.98	4.37	45.00
		0.000	3.55	.0003	0.0000	1.41	0.000	0.00	-1.414	5.42	1.68	4.36	60.00
		0.000	3.54	.0001	0.0000	1.58	0.000	0.00	-1.584	4.57	2.50	4.34	75.00
		0.000	3.52	.0000	0.0000	1.65	0.000	0.00	-1.647	3.65	3.39	4.32	90.00
		0.000	3.50	-.0001	0.0000	1.60	0.000	0.00	-1.599	2.72	4.29	4.30	105.00
		0.000	3.49	-.0003	0.0000	1.44	0.000	0.00	-1.440	1.84	5.14	4.28	120.00
		0.000	3.47	-.0004	0.0000	1.18	0.000	0.00	-1.180	1.08	5.87	4.26	135.00
		0.000	3.46	-.0004	0.0000	.84	0.000	0.00	-.837	.50	6.43	4.25	150.00
		0.000	3.46	-.0005	0.0000	.43	0.000	0.00	-.434	.13	6.79	4.24	165.00
		0.000	3.46	-.0005	0.0000	-.00	0.000	0.00	.000	0.00	6.91	4.24	180.00
		0.000	3.46	-.0005	0.0000	-.43	0.000	0.00	.434	.13	6.79	4.24	195.00
		0.000	3.46	-.0004	0.0000	-.84	0.000	0.00	.837	.50	6.43	4.25	210.00
		0.000	3.47	-.0004	0.0000	-1.13	0.000	0.00	1.180	1.08	5.87	4.26	225.00
		0.000	3.49	-.0003	0.0000	-1.44	0.000	0.00	1.440	1.84	5.14	4.28	240.00
		0.000	3.50	-.0001	0.0000	-1.60	0.000	0.00	1.599	2.72	4.29	4.30	255.00
		0.000	3.52	.0000	0.0000	-1.65	0.000	0.00	1.647	3.65	3.39	4.32	270.00
		0.000	3.54	.0001	0.0000	-1.58	0.000	0.00	1.584	4.57	2.50	4.34	285.00
		0.000	3.55	.0003	0.0000	-1.41	0.000	0.00	1.414	5.42	1.68	4.36	300.00
		0.000	3.56	.0004	0.0000	-1.15	0.000	0.00	1.150	6.15	.98	4.37	315.00
		0.000	3.57	.0005	0.0000	-.81	0.000	0.00	.811	6.70	.45	4.39	330.00
		0.000	3.58	.0005	0.0000	-.42	0.000	0.00	.419	7.05	.11	4.39	345.00
		0.000	3.58	.0005	0.0000	.00	0.000	0.00	-.000	7.17	0.00	4.40	360.00
		0.000	3.58	.0005	0.0000	.42	0.000	0.00	-.419	7.05	.11	4.39	375.00

## MODE=2 ORBIT ADJUST DESIGN

TO PRODUCE A 13.00 SEC PERIOD CHANGE  
AND A -4.30 DEG PERIGEE ROTATION REQUIRES AN INTRACK VELOCITY IMPULSE OF 5.97 METERS PER SEC  
AT A TRUE ANOMALY OF 298.10 DEG

CONVERGENCE OCCURRED IN 3 ITERATIONS

## PREADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTORS

SEMI-MAJOR AXIS = 6756.776000 KM  
ECCENTRICITY = .1810700000E-01  
INCLINATION = 96.97440000 DEG  
ARGUMENT OF PERIGEE = 178.3081000 DEG  
RIGHT ASCENSION OF ASCENDING NODE = 336.0182000 DEG  
TRUE ANOMALY = 0.

X = -3017.923892 KM  
Y = 545.3392087 KM  
Z = 5954.022666 KM  
XDOT = -6.114472035 KM PER SEC  
YDOT = 3.192597566 KM PER SEC  
ZDOT = -3.529684383 KM PER SEC  
RADIUS = 6697.435732 KM  
SPEED = 7.748426893 KM PER SEC

## THRUST PARAMETERS AND INDUCED CHANGES IN ORBITAL PARAMETERS

DVI(M/S)	DVC(M/S)	DVR(M/S)	DA(KM)	DF	DI(DEC)	DW(DEC)	DDM(DEC)	DUI(DEC)	DTA(DEC)	DRA(KM)	DRP(KM)	DP(SEC)	TAI(DEC)
5.969	0.000	0.000	10.59	.0008	0.0000	-4.30	0.000	0.00	4.300	15.88	5.31	13.00	298.10

## POST-ADJUST KEPLERIAN ELEMENTS AND CARTESIAN VECTORS

SEMI-MAJOR AXIS = 6767.369902 KM  
ECCENTRICITY = .1886059852E-01  
INCLINATION = 96.97440000 DEG  
ARGUMENT OF PERIGEE = 174.0079456 DEG  
RIGHT ASCENSION OF ASCENDING NODE = 336.0182000 DEG  
TRUE ANOMALY = 4.300154355

X = -3017.851231 KM  
Y = 545.3260852 KM  
Z = 5953.879268 KM  
XDOT = -6.119578949 KM PER SEC  
YDOT = 3.195177603 KM PER SEC  
ZDOT = -3.531984561 KM PER SEC  
RADIUS = 6697.274440 KM  
SPEED = 7.754568685 KM PER SEC

## DISTRIBUTION

National Aeronautics and Space Administration Scientific and Technical Library Code NHS 22, Rm. BA39 600 Independence Avenue, SW Washington, DC 20546	(2)	Air Force Geophysics Laboratory Hanscom Field Bedford, MA 01731	(2)
Defense Mapping Agency Attn: Mr. Jack Calender Washington, DC 20305	(10)	Goddard Space Flight Center Attn: Dr. David Smith Greenbelt, MD 20771	(1)
Defense Mapping Agency Hydrographic/Topographic Center Attn: Dr. Patrick Fell Washington, DC 20390	(10)	The University of Texas at Austin Attn: Dr. Byron Tapley Austin, TX 78712	(1)
Defense Mapping Agency Aerospace Center Attn: Dr. Robert Ballew St. Louis, MO 63118	(8)	Applied Research Laboratory University of Texas Attn: Dr. Arnold Tucker Austin, TX 78712	(5)
Naval Electronics Systems Command Navy Space Project, PME106 Washington DC 20360	(3)	Physical Sciences Laboratory New Mexico State University Box 3 - PSL Attn: Dan Martin Las Cruces, NM 88003	(3)
Office of Chief of Naval Operations Naval Oceanography Division (NOP-952) Bldg. 1, U. S. Naval Observatory Washington, DC 20390	(2)	Applied Physics Laboratory Johns Hopkins University Johns Hopkins Road Attn: Harold Black Laurel, MD 20810	(3)
Office of Naval Operations Navy Space Systems Division (NOP-943) Washington DC 20350	(2)	Library of Congress Attn: Gift and Exchange Division Washington, DC 20540	(4)
Naval Research Laboratory Attn: Mr. A. Bartholomew Washington, DC 20375	(3)	Local:	
Naval Oceanographic Office Bay St. Louis, MS 39522	(2)	E31 (GIDEP) E411 (Green) E431 F14 K05 K12 K13 K14	(10) (4) (2) (10) (20) (5)
Office of Naval Research Physical Sciences Division 800 N. Quincy St. Arlington, VA 22217	(2)		

**END**

**FILMED**

**9-85**

**DTIC**